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Family of non-equilibrium statistical operators and influence of the past on the present

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Abstract

A family of non-equilibrium statistical operators (NSO) is introduced which differ by the system lifetime distribution over which the quasi-equilibrium (relevant) distribution is averaged. This changes the form of the source in the Liouville equation, as well as the expressions for the kinetic coefficients, average fluxes, and kinetic equations obtained with use of NSO. It is possible to choose a class of lifetime distributions for which thermodynamic limiting transition and to tend to infinity of average lifetime of system is reduced to the result received at exponential distribution for lifetime, used by Zubarev. However there is also other extensive class of realistic distributions of lifetime of system for which and after to approach to infinity of average lifetime of system non-equilibrium properties essentially change. For some distributions the effect of "finite memory" when only the limited interval of the past influence on behaviour of system is observed. It is shown, how it is possible to spend specification the description of effects of memory within the limits of NSO method, more detailed account of influence on evolution of system of quickly varying variables through the specified and expanded form of density of function of distribution of lifetime. The account of character of history of the system, features of its conduct in the past, can have substantial influence on non-equilibrium conduct of the system in a present moment time.

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1 Introduction

One of the most fruitful and successful ways of development of the description of the non-equilibrium phenomena are served by a method of the non-equilibrium statistical operator (NSO) [1, 2]. In work [3] new interpretation of a method of the NSO is given, in which operation of taking of invariant part [1, 2] or

use auxiliary "weight function" (in terminology [4, 5]) in NSO are treated as averaging of quasi-equilibrium statistical operator on distribution of past lifetime of system. This approach adjust with the operations spent in the general theory of random processes, in the renewal theory, and also with the lead Zubarev in work as [2] reception NSO by means of averaging on the initial moment of time.

This treatment of NSO gives to the procedure looking before formal, physical sense of the account of causality and allocation of a real finite time interval in which there is a given physical system. New interpretation leads to various directions of development of NSO method which is compared, for example, with Prigogine's [6] approach, introduction of the operator of internal time, irreversibility at microscopical level.

In Kirkwood's works [7] it was noticed, that the system state in time present situation depends on all previous evolution of the non-equilibrium processes developing it. For example, in real crystals it is held in remembrance their formation in various sorts "defects" (dispositions etc.), reflected in structure of the crystals. Changing conditions of formation of crystals, we can change their properties and create new materials. In works [4, 5] it is specified, that it is possible to use many "weight functions". Any form of density of lifetime distribution gives a chance to write down a source of general view in dynamic Liouville equation which thus becomes, specified Boltzmann and Prigogine [4, 5, 6], and contains dissipative items.

If in Zubarev's works [1, 2] the linear form of a source corresponding limiting exponential distribution for lifetime is used other expressions for density of lifetime distribution give fuller and exact analogues of "integrals of collisions". The obvious account of violation of time symmetry (through finiteness of lifetime, the beginning, the end and irreversibility of a life) is entered. Besides communication with the theory of queues, reliability theory, the management theory, the information theory etc., in offered work the physical consequences connected with fundamental physical problems are reflected.

The formalism follows from the physical matter, for example, from finiteness of lifetime of real physical systems (it is possible to result many examples of problems in which it is necessary to consider systems of the finite sizes with finite lifetime). Generally the description of non-equilibrium systems represents the self-coordinated problem: definition of lifetime through interaction of system with environment [1], dynamics of the operators characterizing non-equilibrium processes, and substitution of found average lifetime in NSO, definition of non-equilibrium physical characteristics, depending on system lifetime.

In work [8] irreversible transfer equations are received in assumptions of coarsening of the distributions, a certain choice of macroscopical variables and the analysis of division of time scales of the description (last circumstance was marked in [9]). Importance and necessity of the analysis of the time scales playing a fundamental role in the description of macroscopical dynamics of system is underlined. Evolution of slow degrees of freedom is described by Markovian equations. Thus the time scale on which observable variables evolve, should be much more time of memory on which the residual effects brought by irrelevant degrees of freedom are considered.

Otherwise effects of memory play an essential role. Memory time is estimated in work [8] for Boltzmann equation. In the present work the consideration subject is made by situations when it is necessary to consider effects of memory. Examples of such situations are given in [8]. We will notice, that the projective methods used in [8], do not consider distribution on lifetime of system (that is noted in [2]) on which method NSO is based.

In work [10] it is shown, in what consequences for non-equilibrium properties of system results change of lifetime distribution of system for systems of the limited volume with finite lifetime. In the present work are considered also infinitely greater systems with infinite average lifetime.

2 New interpretation of NSO

In [3] the Nonequilibrium Statistical Operator introduced by Zubarev [1, 2] rewritten as

$$ln\varrho(t)=\int_0^\infty p_q(u)ln\varrho_q(t-u,-u)du, \quad ln\varrho_q(t,0)=-\Phi(t)-\sum_n F_n(t)P_n;$$

$$ln\varrho_q(t,t_1)=e^{\{-t_1 H/i\hbar\}}ln\varrho_q(t,0)e^{\{t_1 H/i\hbar\}}; \quad \Phi(t)=\ln Sp\exp\{\sum_n F_n(t)P_n\},$$

where H is hamiltonian, $ln\varrho(t)$ is the logarithm of the NSO in Zubarev's form, $ln\varrho_q(t,0)$ is the logarithm of the quasi-equilibrium (or relevant); the first time argument indicates the time dependence of the values of the thermodynamic parameters F_m; the second time argument t_2 in $\varrho_q(t_1,t_2)$ denotes the time dependence through the Heizenberg representation for dynamical variables P_m from which $\varrho_q(t,0)$ can depend [1, 2, 3, 4, 5]. In [3] the auxiliary weight function $p_q(u)=\varepsilon exp\{-\varepsilon u\}$ was interpreted as the probability distribution of lifetime density of a system. Γ is random variables of lifetime from the moment t_0 of its birth till the current moment t; $\varepsilon^{-1}=\langle t-t_0\rangle$; $\langle t-t_0\rangle=\langle\Gamma\rangle$, where $\langle\Gamma\rangle=\int up_q(u)du$ is average lifetime of the system. This time period can be called the time period of getting information about system from its past. Instead of the exponential distribution $p_q(u)$ in (1) any other sample distribution could be taken. This fact was marked in [3] and [4, 5] (where the distribution density $p_q(u)$ is called auxiliary weight function $w(t,t`)$). From the complete group of solutions of Liouville equation (symmetric in time) the subset of retarded "unilateral" in time solutions is selected by means of introducing a source in the Liouville equation

$$\frac{\partial\varrho(t)}{\partial t}+iL\varrho(t)=-\varepsilon(\varrho(t)-\varrho_q(t,0))=J,$$

which tends to zero (value $\varepsilon\to 0$) after thermodynamic limiting transition. Here L is Liouville operator; $iL=-\{H,\varrho\}=\Sigma_k[\frac{\partial H}{\partial p_k}\frac{\partial\varrho}{\partial q_k}-\frac{\partial H}{\partial q_k}\frac{\partial\varrho}{\partial p_k}]$; H is Hamilton function, p_k and q_k are pulses and coordinates of particles; $\{...\}$ is Poisson bracket. In [11] it was noted that the role of the form of the source term in the Liouville equation in NSO method has never been investigated. In [12] it

is stated that the exponential distribution is the only one which possesses the Markovian property of the absence of contagion, that is whatever is the actual age of a system, the remaining time does not depend on the past and has the same distribution as the lifetime itself. It is known [1, 2, 3, 4, 5] that the Liouville equation for NSO contains the source $J = J_{zub} = -\varepsilon[ln\varrho(t) - ln\varrho_q(t, 0)]$ which becomes vanishingly small after taking the thermodynamic limit and setting $\varepsilon \to 0$, which in the spirit of the paper [1] corresponds to the infinitely large lifetime value of an infinitely large system. For a system with finite size this source is not equal to zero. In [5] this term enters the modified Liouville operator and coincides with the form of Liouville equation suggested by Prigogine [6] (the Boltzmann-Prigogine symmetry), when the irreversibility is entered in the theory on the microscopic level. We note that the form of NSO by Zubarev cast in [3] corresponds to the main idea of [6] in which one sets to the distribution function ϱ (ϱ_q in Zubarev's approach) which evolves according to the classical mechanics laws, the coarse distribution function $\widetilde{\varrho}$ ($\varrho(t)$ in the case of Zubarev's NSO) whose evolution is described probabilistically since one perform an averaging with the probability density $p_q(u)$. The same approach (but instead of the time averaging the spatial averaging was taken) was performed in [13].

Besides the Zubarev's form of NSO [1, 2], NSO Green-Mori form [14, 15] is known, where one assumes the auxiliary weight function [4] to be equal $W(t, t') = 1 - (t - t')/\tau; w(t, t') = dW(t, t')/dt' = 1/\tau; \tau = t - t_0$. After averaging one sets $\tau \to \infty$. This situation at $p_q(u = t - t_0) = w(t, t' = t_0)$ coincides with the uniform lifetime distribution. The source in the Liouville equation takes the form $J = ln\varrho_q/\tau$. In [1] this form of NSO is compared to the Zubarev's form.

One could name many (no less than 1000) examples of explicit defining of the function $p_q(u)$. Every definition implies some specific form of the source term J in the Liouville equation, some specific form of the modified Liouville operator and NSO. Thus the family of NSO is defined. If distribution $p_q(u)$ contains n parameters, it is possible to write down n equations for their expression through the parameters of the system. From other side, they are expressed through the moments of lifetime. There is the problem of optimum choice of function $p_q(u)$ and NSO.

3 Modifications to the nonequilibrium description

Let's consider now, what consequences follow from such interpretation of NSO.

3.1 Families of NSO

Setting various distributions for past lifetime of the system, we receive a way of recording of families of NSO. Class of NSO from this family will be connected with a class of distributions for lifetime (taken, for example, from the stochastic theory of storage processes, the theory of queues etc.) and with relaxation

properties of that class of physical systems which is investigated. The general expression for NSO with any distribution

$$ln\varrho(t) = \int_0^\infty p_q(u) ln\varrho_q(t-u,-u)du = \tag{1}$$

$$= ln\varrho_q(t,0) - \int_0^\infty (\int p_q(u)du)\frac{d\ln\varrho(t-u,-u)}{du}du,$$

where integration by parts in time is carried out at $\int p_q(y)dy_{|y=0} = -1$; $\int p_q(y)dy_{|y\to\infty} = 0$; at $p_q(y) = \varepsilon\exp\{-\varepsilon y\}; \varepsilon = 1/\langle\Gamma\rangle$, the expression (1) passes in NSO from [1, 2]. In [12] it is shown, how from random process $X(t)$, corresponding to evolution of quasi-equilibrium system, it is possible to construct set of new processes, introducing the randomized operational time. It is supposed, that to each value $t > 0$ there corresponds a random value $\Gamma(t)$ with the distribution $p_q^t(y)$. The new stochastic kernel of distribution of a random variable $X(\Gamma(t))$ is defined by equality of a kind (1). Random variables $X(\Gamma(t))$ form new random process which, generally speaking, need not to be of Markovian type any more. Each moment of time t of "frozen" quasi-equilibrium system is considered as a random variable $\Gamma(t)$ the termination of lifetime with distribution $p_q^t(y)$. Any moment of lifetime can be with certain probability the last. That the interval $t-t_0 = y$ was enough large (that became insignificant details of an initial condition as dependence on the initial moment t_0 is nonphysical [1, 2]), it is possible to introduce the minimal lifetime $\Gamma_{min} = \Gamma_1$ and to integrate in (1) on an interval (Γ_1, ∞). It results to the change of the normalization density of distribution $p_q(y)$. For example, the function $p_q(y) = \varepsilon\exp\{-\varepsilon y\}$ will be replaced by $p_q(y) = \varepsilon\exp\{\varepsilon\Gamma_1 - \varepsilon y\}, y \geq \Gamma_1; p_q(y) = 0, y < \Gamma_1$. The under limit of integration in (1) by $\Gamma_1 \to 0$ is equal 0. It is possible to choose $p_q(y) = Cf(y), y < t_1; p_q(y) = \varepsilon\exp\{-\varepsilon y\}, y \geq t_1; C = (1 - exp\{-\varepsilon t_1\})/(\int_0^{t_1} f(y)dy)$. The function $f(y)$ can be taken from models of the theory of queues, the stochastic theory of storage and other sources estimating the lifetime distribution for small times (for example [16, 17, 18, 27, 29]). The value t_1 can be found from results of work [18]. It is possible to specify many concrete expressions for lifetime distribution of system, each of which possesses own advantages. To each of these expressions there corresponds own form of a source in Liouville equation for the nonequilibrium statistical operator. In general case any functions $p_q(u)$ the source is:

$$J = p_q(0)\ln\varrho_q(t,0) + \int_0^\infty \frac{\partial p_q(y)}{\partial y}(ln\varrho_q(t-y,-y))dy \tag{2}$$

(when values $p_q(0)$ disperse, it is necessary to choose the under limit of integration equal not to zero, and Γ_{min}). Such approach corresponds to the form of dynamic Liouville equation in the form of Boltzmann-Bogoliubov-Prigogine [4, 5, 6], containing dissipative items.

Thus the operations of taking of invariant part [1], averaging on initial conditions [2], temporary coarse-graining [7], choose of the direction of time [4, 5], are replaced by averaging on lifetime distribution.

The physical sense of averaging on introduced lifetime distribution of quasi-equilibrium system as it was already marked, consists in the obvious account of infringement of time symmetry and loss (reduction accessible) the information connected with this infringement, that is shown in occurrence the value of average of entropy production $\langle \Delta S(t) \rangle$ not equal to zero, obviously reflecting fluctuation-dissipative processes at the real irreversible phenomena in non-equilibrium systems. The correlations received in the present section generalize results of statistical non-equilibrium thermodynamics [1, 2] and information statistical thermodynamics [4, 5] as instead of weight function of a form $\varepsilon \exp\{\varepsilon t'\}$ contain density of probability of lifetime of quasi-equilibrium system which as it was already marked, can not coincide with exponential distribution (in the latter case it coincides with weight function from [1, 2]). For example, for system with n classes of ergodic states limiting exponential distribution is replaced with the general Erlang. In research of lifetimes of complex systems it is possible to involve many results of the theory of reliability, the theory of queues, the stochastic theory of storage processes, theory of Markov renewal, the theory of semi-Markov processes etc.

It is essentially that $\varepsilon \neq 0$. The thermodynamic limiting transition is not performed, and actually important for many physical phenomena dependence on the size of system are considered. We assume ε and $\langle \Gamma \rangle$ to be finite values. Thus the Liouville equation for $\varrho(t)$ contains a finite source. The assumption about finiteness of lifetime breaks temporary symmetry. And such approach (introduction $p_q(y)$, averaging on it) can be considered as completing the description of works [1, 2].

In work [18] lifetimes of system are considered as the achievement moments by the random process characterizing system, certain border, for example, zero. In [18] are received approached exponential expressions for density of probability of lifetime, accuracy of these expressions is estimated. In works [19, 20] lifetimes of molecules are investigated, the affinity of real distribution for lifetime and approached exponential model is shown. It is possible to specify and other works (for example [21, 22, 23]) where physical appendices of concept of lifetime widely applied in such mathematical disciplines, as reliability theory, the theory of queues and so forth (under names non-failure operation time, the employment period, etc.) are considered. In the present section lifetime joins in a circle of the general physical values, acting in an estimation or management role (in terminology of the theory of the information [24]) for the quasi-equilibrium statistical operator that allows to receive the additional information on system. In [24] it is noticed, that three disciplines grow together: statistical thermodynamics, Shennon's theory of the information and the theory of optimum statistical decisions. Accordingly, all correlations written down in the present work can be interpreted in terms of the theory of the information or the theory of optimum statistical decisions [25].

Let's notice, that in a case when value $d \ln \varrho_q(t-y, -y)/dy$ (the operator of entropy production σ [1]) in the second item of the right part (1) does not depend from y and is taken out from under integral on y, this second item becomes $\sigma\langle\Gamma\rangle$, and expression (1) does not depend on form of function $p_q(y)$. There is it, for example, at $\varrho_q(t) \sim \exp\{-\sigma t\}, \sigma = const$. In work [26]) such

distribution is received from a principle of a maximum of entropy at the set of average values of fluxes.

3.2 Physical sense of distributions for past lifetime of system

As is known (for example, [16], [18], [27]), exponential distribution for lifetime

$$p_q(y) = \varepsilon \exp\{-\varepsilon y\}, \tag{3}$$

used in Zubarev's works [1, 2], is limiting distribution for lifetime, fair for large times. It is marked in works [1, 2] where necessity of use of large times connected with damping of nonphysical initial correlations. Thus, in works [1, 2] the thermodynamic result limiting and universal is received, fair for all systems. It is true in a thermodynamic limit, for infinitely large systems. However real systems have the finite sizes. Therefore essential there is use of other, more exact distributions for lifetime. In this case the unambiguity of the description peculiar to a thermodynamic limit [28] is lost.

For NSO with Zubarev's function (3) the value enter in second item

$$-\int p_q(y)dy = \exp\{-\varepsilon y\} = 1-\varepsilon y+(\varepsilon y)^2/2-... = 1-y/\langle\Gamma\rangle+y^2/2\langle\Gamma\rangle^2-... \tag{4}$$

Obviously that to tend to infinity of average lifetime, $\langle\Gamma\rangle \to \infty$, correlation (4) tends to unity.

Besides exponential density of probability (3), as density of lifetime distribution Erlang distributions (special or the general), gamma distributions etc. (see [16, 17]), and also the modifications considering subsequent composed asymptotic of the decomposition [27] can be used. General Erlang distributions for n classes of ergotic states are fair for cases of phase transitions or bifurcations. For $n = 2$ general Erlang distribution looks like $p_q(y) = \theta\rho_1 \exp\{-\rho_1 y\} + (1-\theta)\rho_2 \exp\{-\rho_2 y\}, \theta < 1$. Gamma distributions describe the systems which evolution has some stages (number of these stages coincides with gamma distribution order). Considering real-life stages in non-equilibrium systems (chaotic, kinetic, hydrodynamic, diffusive and so forth), it is easy to agree, first, with necessity of use of gamma distributions of a kind

$$p_q(y) = \varepsilon(\varepsilon y)^{k-1} \exp\{-\varepsilon y\}/\Gamma(k) \tag{5}$$

($\Gamma(k)$ is gamma function, at $k = 1$ we receive distribution (3)), and, secondly, - with their importance in the description of non-equilibrium properties.

More detailed description $p_q(u)$ in comparison with limiting exponential (3) allows to describe more in detail real stages of evolution of system (and also systems with small lifetimes). Each from lifetime distributions has certain physical sense. In the theory of queues, for example [34], to various disciplines of service there correspond various expressions for density of lifetime distribution. In the stochastic theory of storage [34], to these expressions there correspond various models of an exit and an input in system.

The value ε without taking into account of dissipative effects can be defined, for example, from results of work [18]. The value ε is defined also in work [3] through average values of operators of entropy and entropy production, flows of entropy and their combination.

How was already marked, it is possible to specify very much, no less than 1000, expressions for the distributions of past lifetime of the system. Certain physical sense is given to each of these distributions. To some class functions of distributions, apparently, some class of the physical systems corresponds, the laws of relaxation in which answer this class of functions of distributions for lifetime.

3.3 Influence of the past on non-equilibrium properties

A). Expressions for average fluxes.

In [12] by consideration of the paradox connected with a waiting time, the following result is received: let $X_1 = S_1; X_2 = S_2 - S_1; ...$ are mutually independent also it is equally exponential the distributed values with average $1/\varepsilon$. Let $t > 0$ is settled, but it is any. Element X_k, satisfying to condition $S_{k-1} < t \leq S_k$, has density $\nu_t(x) = \varepsilon^2 x \exp\{-\varepsilon x\}, 0 < x \leq t; \nu_t(x) = \varepsilon(1 + \varepsilon x) \exp\{-\varepsilon x\}, x > t$. In Zubarev's NSO [1, 2] the value of lifetime to a present moment t, belonging lifetime X_k, influence of the past on the present is considered. Therefore the value $p_q(u)$ should be chosen not in the form of exponential distribution (3), and in a form

$$p_q(y) = \varepsilon^2 y \exp\{-\varepsilon y\}, \tag{6}$$

that in the form of gamma distribution (5) at $k = 2$. In this case distribution (6) coincides with special Erlang distribution of order 2 [16], when refusal (in this case - the moment t) comes in the end of the second stage [16], the system past consists of two independent stages. Function of distribution is equal $P_q(x) = 1 - \exp\{-\varepsilon x\} - \varepsilon x \exp\{-\varepsilon x\}, p_q(u) = dP_q(u)/du$, unlike exponential distribution, when $P_q(x) = 1 - \exp\{-\varepsilon x\}$. The behaviour of these two densities of distribution of a form (3) and (6) essentially differs in a zero vicinity. In case of (6) at system low probability to be lost at small values y, unlike exponential distribution (3) where this probability is maximal. Any system if has arisen, exists any minimal time, and it is reflected in distribution (6).

In work [10] it is shown, in what differences from Zubarev's distribution (1) with exponential distribution of lifetime (3) results gamma distribution (5), (6) use. Additional items in NSO, in integral of collisions of the generalized kinetic equation, in expressions for average fluxes and self-diffusions coefficient are considered. The same in [10] is done and for special Erlang distribution $k = 2, 3, 4..., n, P_q(x) = 1 - \exp\{-\varepsilon x\}[1 + \varepsilon x/1! + ... + (\varepsilon x)^{k-1}/(k-1)!]; \varepsilon = k/\langle\Gamma\rangle$. For distributions (5), (6) is correct correlation (4), value $-\int p_q(u)du \to 1$ by $\langle\Gamma\rangle \to \infty$.

Thus the multi-stage model of the past of system is introduced. Non-equilibrium processes usually proceed in some stages, each of which is characterized by the time scale. In distribution (6) the account of two stages,

possibly, their minimal possible number is made. Other distributions can describe any other features of the past. Corresponding additives will be included into expressions for fluxes, integral of collisions, kinetic coefficients. Besides special Erlang distributions with whole and specified values $k = n$, that does not deduce us from set of one-parametrical distributions, the general already two-parametrical gamma distribution where the parameter k can accept any values can be used. In this case $\langle\Gamma\rangle = k/\varepsilon$. The situation (formally), when $k < 1$ is possible. Then sources will tends to infinity, as $(t-t_0)^{k-1}_{|t\to t_0} \to \infty$ at $k < 1$. This divergence can be eliminated, having limited from below the value $t-t_0$ of minimal lifetime value Γ_{min}, having replaced the under zero limit of integration on Γ_{min}. Then to expression for a source (2) it is added item $[(\varepsilon\Gamma_{min})^{k-1}/\Gamma(k)]\varepsilon\exp\{-\varepsilon\Gamma_{min}\}\ln\varrho_q(t-\Gamma_{min}, -\Gamma_{min})$.

B). Entropy production.

Expressions for average entropy production received, for example, in [4], also depend from w (or $p_q(u)$ - in designations [3] and this work), i.e. on the chosen form of density of probability of distribution of the past of system. So, for average entropy production $\overline{\sigma} = d\overline{S}/dt$ in work [4] expression $\overline{\sigma}(t) = \Sigma_{k=1}^{\infty}\int_{t_0}^{t} dt' W(t,t')(\sigma(z|t,0); \sigma(z|t',t'-t)|t)^k$ is received, where z are points in phase space , $\sigma(z|t',t'-t) = -d\ln\varrho_q(z|t',t'-t)/dt'$, $(\sigma(z|t,0); \sigma(z|t',t'-t)|t)^k = (k!)^{-1}\int dz\sigma(z|t,0)\sigma(z|t',t'-t)\int_{t_0}^{t} dt_1 W(t,t_1)\sigma(z|t_1,t_1-t)...\int_{t_0}^{t} dt_{k-1}W(t,t_{k-1})$ $\sigma(z|t_{k-1},t_{k-1}-t)\varrho_q(z|t,0), w(t,t') = dW(t,t')/dt'$ is "auxiliary weight function" (in terminology [4]); $w(t,t')$ it is designated above as $p_q(u); w(t,t') = p_q(u = t-t')$. For the limiting exponential distribution (3) used in Zubarev's NSO, $W(t,t') = \exp\{\varepsilon(t'-t)\}$.

4 Systems with infinite lifetime

Above, as well as in work [10], additives to NSO in the Zubarev's form are received for systems of the finite size, with finite lifetime. We will show, as for systems with infinite lifetime, for example, for systems of infinite volume, after thermodynamic limiting transition, the same effects, which essence in influence of the past of system, its histories, on its present non-equilibrium state are fair.

In work [10] it is shown, as changes in function $p_q(u)$ influences on non-equilibrium descriptions of the system. But for those distributions $p_q(u)$, which are considered in [10] (gamma-distributions, (5), (6)) the changes show up only for the systems of finite size with finite lifetimes. Additions to unit in equation (4) becomes vanishingly small to tend to infinity of sizes of the system and its average lifetime, as in the model distribution (3) used in Zubarev's NSO (1). For the systems of finite size and the exponential distribution results to nonzero additions in expression (1). Thus, these additions to NSO and proper additions to kinetic equations, kinetic coefficients and other non-equilibrium descriptions of the system, are an effect finiteness of sizes and lifetime of the system, not choice of distribution of lifetime of the system. We will find out, whether there are distributions of lifetime of system for which and for the infinitely large systems with infinitely large lifetime an additional contribution to NSO differs from Zubarev'a NSO.

4a). Let's consider in quality $p_q(u)$ distribution of a form

$$p_q(u) = \frac{ku^{k-1}\rho^k}{[1+(u\rho)^k]^2},$$

received in work [17], where $k = 1/\tau, \nu = -\log\rho, \tau$ and ν are parameters of scale and shift of logistical distribution $f(x) = \tau^{-1}\exp[(x-\nu)/\tau]/\{1+\exp[(x-\nu)/\tau]\}^2$. In the correlatioon (1) the value $\int p_q(u)du = -1/[1+(u\rho)^k]$ appears. Average value of lifetime is equal

$$\langle\Gamma\rangle = \int_0^\infty up_q(u)du = \rho^{-1}B(1/(k+1), 1-1/k), \tag{7}$$

$$\langle\Gamma\rangle^2 = \int_0^\infty u^2 p_q(u)du = \rho^{-2}B(2/(k+1), 1-2/k),$$

where $B(,)$ is beta function [32].

The value $\langle\Gamma\rangle$ in (7) to tend to infinity at $a)\rho = 0, b)k = 1$. Ratio of the second moment toward the square of the first is equal

$$\frac{\langle\Gamma^2\rangle}{\langle\Gamma\rangle^2} = \frac{B(2/(k+1), 1-2/k)}{B^2(1/(k+1), 1-1/k)}. \tag{8}$$

Expression (8) becomes vanishingly small with $k \to 1$. Correlation (1) for this distribution takes the form

$$\ln\varrho(t) = \int_0^\infty p_q(y)\ln\varrho_q(t-y,-y)dy = ln\varrho_q(t,0) +$$
$$+\int_0^\infty (\frac{1}{[1+(u\rho)^k]})(\frac{d\ln\varrho_q(t-u,-u)}{du})du.$$

At $k \to 1$ and finite values ρ we have a difference from the zero of additions to unit in expansion

$$\ln\varrho(t) = ln\varrho_q(t,0)+\int_0^\infty (1-(u\rho)^k+(u\rho)^{2k}-(u\rho)^{3k}+...)(d\ln\varrho_q(t-u,-u)/du)du.$$

But value k it is possible to define from correlation (8), and, if (8) is finite value not equal to the zero then $k \neq 1$. There is $\rho \to 0$, when additions becomes vanishingly small, as in the case of (4) of Zubarev's NSO.

4b). Pareto distribution [17]

$$p_q(u) = \frac{ka^k}{[u+a]^{k+1}} = \frac{k(k/\rho_0)^k}{[u+(k/\rho_0)]^{k+1}}, \quad \rho_0 = k/a. \tag{9}$$

This distribution is received in [17], as complex exponential distribution. It is supposed, that intensity ρ of exponential distribution $f(u) = \rho\exp\{-\rho u\}$ represents random variable P with distribution $f_P(\rho)$. Then

$$f_T(u) = p_q(u) = \int_0^\infty \rho\exp\{-\rho u\}f_P(\rho)d\rho.$$

If to be set for function $f_P(\rho)$ by gamma distribution with density

$$f_P(\rho) = a^k \rho^{k-1} \exp\{-a\rho\}/\Gamma(k) \tag{10}$$

that we receive distribution (9), Pareto distribution. From (9) it is obtained:

$$\int p_q(u)du = -\frac{a^k}{(u+a)^k} = -[1 - \frac{uk}{a} + \frac{u^2 k(k+1)}{2a^2} - \frac{u^3 k(k+1)(k+2)}{6a^3} + ...];$$

$$\langle\Gamma\rangle = a/(k-1), \quad k \geq 1.$$

We will consider two cases:

a). The parameter k is fixed, $a = \langle\Gamma\rangle(k-1)$. Then, as in case of exponential (3) or gamma distributions (5) for $p_q(u)$ additives to NSO are proportional to $1/\langle\Gamma\rangle$, they becomes vanishingly small with $\langle\Gamma\rangle \to \infty$ as in (4).

b). The parameter a is fixed. Then $k = 1 + a/\langle\Gamma\rangle$; $-\int p_q(u)du = a^k/(u+a)^k = 1 - (u/a)(1 + a/\langle\Gamma\rangle) + (u^2/2a^2)(1 + a/\langle\Gamma\rangle)(2 + a/\langle\Gamma\rangle) - (u^3/6a^3)(1 + a/\langle\Gamma\rangle)(2 + a/\langle\Gamma\rangle)(3 + a/\langle\Gamma\rangle) + ... \to 1/(1 + u/a)$, $\langle\Gamma\rangle \to \infty, k \to 1$. We will mark that the second moment of Pareto distribution does not exist at $k \leq 2$ [29]. And at $\langle\Gamma\rangle \to \infty, k \to 1$.

Pareto distribution (9) corresponds to Tsallis distribution [30]

$$p_q(u) = \frac{1}{Z[1+\beta(q-1)u]^{1/(q-1)}}; \quad k+1 = \frac{1}{(q-1)}; \quad q = \frac{(k+2)}{(k+1)}; \quad a = \frac{(k+1)}{\beta}.$$

In Tsallis method averaging is conducted on distribution

$$p^q(u) = \frac{(k/a)^{(k+2)/(k+1)}}{[1+u/a]^{k+2}}.$$

Then

$$\langle\Gamma\rangle = \frac{\int_0^\infty u p^q(u)du}{\int_0^\infty p^q(u)du} = \frac{a}{k}; \quad \frac{D}{\langle\Gamma^2\rangle} = \frac{(k+1)}{(k-1)}; \quad D = \langle\Gamma^2\rangle - \langle\Gamma\rangle^2;$$

$$k = \frac{(s+1)}{(s-1)}; \quad s = \frac{D}{\langle\Gamma^2\rangle}; \quad -\int p_q(u)du = \frac{a^k}{(u+a)^k} =$$

$$= 1 - \frac{uk}{a} + \frac{u^2 k(k+1)}{2a^2} - \frac{u^3 k(k+1)(k+2)}{6a^3} + ... =$$

$$= 1 - \frac{u}{\langle\Gamma\rangle} + \frac{u^2 2s}{2!(s+1)\langle\Gamma\rangle^2} - \frac{u^3 2s(3s-1)}{3!(s+1)^2\langle\Gamma\rangle^3} + ... \to 1, \quad \langle\Gamma\rangle \to \infty,$$

as in (4). By finite values of ratio $s = D/\langle\Gamma^2\rangle$ limiting behaviour of NSO is same, as in (4) and [10].

4c). Let's consider one more distribution for $p_q(u)$, connected with degree laws. In work [12] by consideration of the renewal theory it is received distribution for length of an interval $t - S_{N_t}$, where S_N are the renewal moments;

$S_{N_t} < t < S_{N_t+1}$. If to interpret the renewal moments as a birth and destruction of system the interval $t - S_{N_t}$ represents time of past life of system, the value $t - t_0$ from (1). In [12] it is shown, that $P\{t - S_{N_t} > x, S_{N_t+1} - t > y\} \to \mu^{-1}\int_{x+y}^{\infty}(1 - F(s))ds$, where $F(s)$ is distribution of random variable T_i from sum $S_n = S_0 + T_1 + ... + T_n, \mu = \int_0^{\infty}(1 - F(s))ds = \int_0^{\infty} sF(s)ds$. At large s: $1 - F(s) \sim s^{-\alpha}L(s), 0 < \alpha < 2, L(s)$ is slowly varying function. Thus, distribution $p_q(u)$ at large values t and $x + y$ looks like $u^{-\alpha}$. We will break all time interval on two parts and we will describe at large times function $p_q(u)$ degree dependence, and on small times we set $p_q(u)$ gamma function of the form (5), (10) with $k = 2$, i.e (6). Thus,

$$p_q(u) = \{ \begin{array}{ll} \varepsilon^2 u \exp\{-\varepsilon u\}, & u < c; \\ bu^{-\alpha}, & u \geq c, \end{array} \tag{11}$$

where c is some value of time. From a normalization condition of distribution (11) we find, that at $1 < \alpha < 2$,

$$b = \frac{-(1-\alpha)\varepsilon \exp\{-\varepsilon c\}(c + 1/\varepsilon)}{c^{-\alpha+1}},$$

But value $\langle\Gamma\rangle$ disperses. Therefore we will be limited not to an infinite limit of integration on time, and some limiting value of lifetime Γ_m. In this case the normalization condition gives value

$$b = \frac{-(1-\alpha)\varepsilon \exp\{-\varepsilon c\}(c + 1/\varepsilon)}{(\Gamma_m^{-\alpha+1} - c^{-\alpha+1})},$$

and for average value of lifetime we receive expression

$$\langle\Gamma\rangle = \frac{2}{\varepsilon} + \exp\{-\varepsilon c\}[\frac{(1-\alpha)\varepsilon(c + 1/\varepsilon)(\Gamma_m^{-\alpha+2} - c^{-\alpha+2})}{(2-\alpha)(\Gamma_m^{-\alpha+1} - c^{-\alpha+1})} - \frac{(c^2\varepsilon^2 + 2c\varepsilon + 2)}{\varepsilon}]. \tag{12}$$

If to fix parameters Γ_m, c, ε, dependence $\varepsilon(\langle\Gamma\rangle)$, defined from (12), will be positive for a limiting case interesting us

$$\Gamma_m \to \infty, \quad \langle\Gamma\rangle \to \infty, \quad r_m = lim\langle\Gamma\rangle/\Gamma_m$$

at enough small values r_m. For example, in case of extension $\exp\{-\varepsilon c\}$ in series and restrictions of cubic items, we receive, that

$$\varepsilon^2 \approx \frac{(\langle\Gamma\rangle - g)}{c^2(c/3 - g/2)}; \quad g = \frac{(1-\alpha)(\Gamma_m^{-\alpha+2} - c^{-\alpha+2})}{(2-\alpha)(\Gamma_m^{-\alpha+1} - c^{-\alpha+1})}.$$

In a limiting case

$$\Gamma_m \to \infty, \quad \langle\Gamma\rangle \to \infty, \quad \varepsilon^2 = \frac{2(g_1 - r_m)}{c^2 g_1}, \quad g_1 = \frac{(1-\alpha)}{(\alpha-2)} > 0.$$

That was $\varepsilon^2 > 0$, should be r_m less g_1. The ratio r_m is finite at $\Gamma_m \sim u^{-\alpha+2}$ as the value $\langle\Gamma\rangle$ disperses as $u^{-\alpha+2}$ at $u \to \infty$. Thus in a limiting case the value ε

remains finite, and all additives entering in $p_q(u)$ and in additional expressions to NSO all additives are finite too unlike (4).

If to fix parameters Γ_m, c, ε, defining dependence α from $\langle\Gamma\rangle$ from (12) it is received, that at finite values c and ε, at $\Gamma_m \to \infty, \langle\Gamma\rangle \to \infty, r_m = lim\langle\Gamma\rangle/\Gamma_m$

$$\alpha = \frac{(m + 2r_m)}{(m + r_m)}, \quad m = \exp\{-\varepsilon c\}(\varepsilon c + 1).$$

It is possible to consider and other limiting cases, and other distributions for $p_q(u)$. But the general conclusion consists in that, as for infinitely large systems and infinitely large lifetimes the task of realistic distributions for time the lived of system a life changes a non-equilibrium state of system. The account of character of history of system, features of its behaviour in the past, can make essential influence on non-equilibrium behaviour of system in present time situation.

4d). One more distribution for $p_q(u)$ can be received from results of works [31], integrating distribution $P(E,\Gamma)$ on E

$$p_q(u) = \exp\{-\gamma u\}(1 - c\exp\{-\gamma u\})^{-1/(q-1)}; \quad c = (q-1)\gamma a q^{-1}; \qquad (13)$$

$$a = (\exp\{\beta PV\} - 1)^{-1}; \quad \beta = 1/kT,$$

where $V = \Delta$ is volume of metastable area, P is pressure, T is temperature [31]. The normalization of distribution (13) and its moments are expressed through incomplete beta-function [32]. For example, average value of lifetime is equal

$$\gamma a\langle\Gamma\rangle = \frac{a^{-1}\Gamma^2(1/a)_3F_2(a^{-1}, a^{-1}, 1/(q-1); 1 + a^{-1}, 1 + a^{-1}, c)}{{}_2F_1(a^{-1}, 1/(q-1); 1 + a^{-1}, c)} =$$

$$= \frac{a^{-1}\Gamma^2(a^{-1})(1 + \frac{a\gamma}{q(1+a)^2} + \frac{a^2\gamma^2}{2q(1+2a)^2} + ...)}{(1 + \frac{a\gamma}{q(1+a)} + \frac{a^2\gamma^2}{2q(1+2a)} + ...)},$$

$\Gamma(a^{-1})$ is gamma function, ${}_nF_m$ is hypergeometrical function [32]. The ratio of a dispersion to a square of average value is equal

$$D/\langle\Gamma\rangle^2 =$$

$$= \frac{2(a)_4F_3(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}, \frac{1}{(q-1)}; 1 + \frac{1}{a}, 1 + \frac{1}{a}, 1 + \frac{1}{a}; c)_2F_1(\frac{1}{a}, 1/(q-1); 1 + \frac{1}{a}; c)}{\Gamma(\frac{1}{a})_3F_2^2(\frac{1}{a}, \frac{1}{a}, \frac{1}{(q-1)}; 1 + \frac{1}{a}, 1 + \frac{1}{a}; c)}.$$

If to assume a little values γ and γa, and to be limited linear items,

$$\frac{\gamma}{a} = (\frac{s\Gamma(a^{-1})}{2a} - 1)\frac{(1+a)^3}{a^3}; \quad D/\langle\Gamma\rangle^2 = s \approx \frac{2a(1 + \frac{\gamma a^3}{q(1+a)^3})}{\Gamma(1/a)};$$

$$\frac{\gamma a^2\langle\Gamma\rangle}{\Gamma^2(1/a)} \approx 1-\frac{a^2\gamma}{(1+a)^2q}=1-\frac{(1+a)(\frac{s\Gamma(1/a)}{2a}-1)}{a}.$$

From here $\gamma\to 0$ at $\langle\Gamma\rangle\to\infty$ at finite values a, and we receive Zubarev's result (1), (3), (4) and [10], when additives to NSO becomes vanishingly small with $\langle\Gamma\rangle\to\infty$. The same gives also square-law approach on γa.

Let's consider now lifetime distributions of a various form to various time scales as (11). In work [10] it was marked that during evolution the system passes various stages (kinetic, hydrodynamic, etc.). Lifetime can end at any stage. At different stages the functions ϱ_q accept a various kind. Therefore and expression for NSO (1) becomes complicated.

4e). Let's consider distribution of kind

$$p_q(u)=\Big\{\begin{array}{l}\varepsilon\exp\{-\varepsilon u\},\quad u<c;\\ \frac{bka^k}{(u+a)^{k+1}},\quad u\geq c,\end{array} \tag{14}$$

combining exponential distribution (3) for small times and fractional Pareto distribution (9) for $u\geq c$. From a normalization condition $1=\varepsilon\int_0^c exp\{-\varepsilon u\}du+b\int_c^\infty\frac{kdu}{a(1+u/a)^{k+1}}$ it is found a normalizing constant, $b=c^k exp\{-\varepsilon c\}$, and then the first and second moments of lifetime:

$$\langle\Gamma\rangle=\frac{1}{\varepsilon}[1-e^{-\varepsilon c}(1+\varepsilon c)]-e^{-\varepsilon c}a[1+\frac{kc}{(1-k)}];$$

$$\langle\Gamma^2\rangle=\frac{2}{\varepsilon^2}[1-e^{-\varepsilon c}(1+\varepsilon c+\frac{\varepsilon^2c^2}{2})]-e^{-\varepsilon c}a^2[\frac{kc^2}{(2-k)}-\frac{2kc}{(1-k)}-1].$$

For Pareto distribution at $k<2$ the second moment does not exist [29]. The value $\langle\Gamma\rangle\to\infty$ with $k\to 1$ from above. Limiting transition $k\to 1$ should be spent after thermodynamic limiting transition. The values ε, c, a remains finite. Then

$$-\int p_q(u)du=\Big\{\begin{array}{l}\exp\{-\varepsilon u\},\quad u<c;\\ \frac{c^k\exp\{-\varepsilon c\}}{(1+u/a)^k},\quad u\geq c\end{array};$$

$$-\int p_q(u)du\ \begin{array}{c}\Rightarrow\\ k\to 1\end{array}\ \Big\{\begin{array}{l}(1-\varepsilon u+...),\quad u<c;\\ c\exp\{-\varepsilon c\}(1-u/a+...),\quad u\geq c.\end{array}$$

Thus, in this case additives to unit and to Zubarev's NSO are not equal to zero and for infinitely large systems with $\langle\Gamma\rangle\to\infty$. Uncertain there are values of parameters ε, c, a. As function $p_q(u)$ it is possible to choose both more simple and more difficult functions.

4f). If to choose

$$p_q(u)=\Big\{\begin{array}{l}\varepsilon\exp\{-\varepsilon u\},\quad u<c;\\ b,\quad u\geq c,\end{array}$$

that $b=\frac{\exp\{-\varepsilon c\}}{(\Gamma_m-c)}$, integral in limits from 0 to Γ_m , but not from 0 to ∞.

$$\langle\Gamma\rangle=\frac{1}{\varepsilon}[1-e^{-\varepsilon c}(1+\varepsilon c)]+e^{-\varepsilon c}\frac{(\Gamma_m+c)}{2};$$

$$\langle\Gamma\rangle \to \infty, \quad \Gamma_m \to \infty, \quad \frac{\langle\Gamma\rangle}{\Gamma_m} \to \frac{e^{-\varepsilon c}}{2};$$

$$\langle\Gamma^2\rangle = \frac{2}{\varepsilon^2}[1 - e^{-\varepsilon c}(1 + \varepsilon c + \frac{\varepsilon^2 c^2}{2})] + \frac{e^{-\varepsilon c}}{3}(\Gamma_m^2 + \Gamma_m c + c^2);$$

$$\frac{\langle\Gamma^2\rangle}{\Gamma_m^2} \to \frac{e^{-\varepsilon c}}{3} = \frac{2\langle\Gamma\rangle}{3\Gamma_m}, \quad \Gamma_m \to \infty.$$

If after thermodynamic limiting transition $\Gamma_m \to \infty$, then $b \to 0$,

$$p_q(u) = \left\{ \begin{array}{ll} \varepsilon \exp\{-\varepsilon u\}, & u < c; \\ \frac{\exp\{-\varepsilon c\}}{(\Gamma_m - c)}, & u \geq c; \end{array} \right.$$

$$p_q(u) \Rightarrow \left\{ \begin{array}{ll} \exp\{-\varepsilon u\}, & u < c; \\ 0, & u \geq c. \end{array} \right.$$

The value ε is finite, and additives to unit are not equal to zero. But in this case the effect of "finite memory", limited on time by the size c, is observed.

4g). For

$$p_q(u) = \left\{ \begin{array}{ll} \varepsilon_1^2 u \exp\{-\varepsilon_1 u\}, & u < c; \\ b\varepsilon_2 \exp\{-\varepsilon_2 u\}, & u \geq c, \end{array} \right. \tag{15}$$

we write down from a normalization condition $b = \exp\{-\varepsilon_1 c\}(1 + \varepsilon_1 c)$ and the two first moment

$$\langle\Gamma\rangle = \frac{1}{\varepsilon_2} e^{-\varepsilon_1 c}(1 + \varepsilon_1 c) e^{-\varepsilon_2 c}(1 + \varepsilon_2 c) + \frac{2}{\varepsilon_1}[1 - e^{-\varepsilon_1 c}(1 + \varepsilon_1 c + \frac{(\varepsilon_1 c)^2}{2})];$$

$\langle\Gamma\rangle \to \infty$ by $\varepsilon_2 \to 0$;

$$\langle\Gamma^2\rangle = \frac{6}{\varepsilon_1^2}[1 - e^{-\varepsilon_1 c}(1 + \varepsilon_1 c + \frac{(\varepsilon_1 c)^2}{2} + \frac{(\varepsilon_1 c)^3}{6}] +$$

$$+ e^{-\varepsilon_1 c} \frac{2}{\varepsilon_2^2} e^{-\varepsilon_2 c}(1 + \varepsilon_1 c)(1 + \varepsilon_2 c + \frac{(\varepsilon_2 c)^2}{2}).$$

Then

$$-\int p_q(u)du = \left\{ \begin{array}{ll} (1 + \varepsilon_1 u)e^{-\varepsilon_1 u}, & u < c; \\ (1 + \varepsilon_1 c)e^{-\varepsilon_1 c} e^{-\varepsilon_2 u}, & u \geq c; \end{array} \right.$$

$$-\int p_q(u)du \underset{\varepsilon_2 \to 0, \langle\Gamma\rangle \to \infty}{\Rightarrow} \left\{ \begin{array}{ll} 1 - \frac{(\varepsilon_1 u)^2}{2} + ..., & u < c; \\ (1 + \varepsilon_1 c)e^{-\varepsilon_1 c}, & u \geq c. \end{array} \right.$$

Additives to unit to tend to infinity of average lifetime are not equal to zero.

4h). For

$$p_q(u) = \left\{ \begin{array}{ll} \varepsilon^2 u \exp\{-\varepsilon u\}, & u < c; \\ \frac{b \exp\{-\gamma u\}}{[1 + (q-1)a\gamma \exp\{-\gamma a u\}/q]^{1/(q-1)}}, & u \geq c, \end{array} \right.$$

combination of (6) and (13),

$$b=\frac{\gamma a[(q-1)\gamma a/q]^{1/a}[1-\exp\{-\varepsilon c\}(1+\varepsilon c)]}{B_{(1-p,1)}(1-1/(q-1),1/a)};\quad p=(q-1)a\gamma\exp\{-\gamma ac\}/q;$$

$$\langle\Gamma\rangle=\frac{2}{\varepsilon}[1-\exp\{-\varepsilon c\}(1+\varepsilon c+\frac{(\varepsilon c)^2}{2})]+$$

$$+[1-\exp\{-\varepsilon c\}(1+\varepsilon c)]\frac{\Gamma^2(1/a)_3F_2(1/a,1/a,1/(q-1);1+1/a,1+1/a;p)}{(a^2\gamma)_2F_1(1/a,1/(q-1);1+1/a;p)};$$

$B_{(1-p,1)}(1-1/(q-1),1/a)$ is incomplete beta function; $\langle\Gamma\rangle\to\infty$ when $\gamma\to 0$, as in 4d), (13), $b\to 0$ when $\gamma\to 0$, after of thermodynamic limiting transition, and

$$p_q(u)du\ \underset{\gamma\to 0}{\Rightarrow}\ \left\{\begin{array}{l}\varepsilon^2 u\exp\{-\varepsilon u\},\quad u<c;\\ 0,\quad u\geq c.\end{array}\right.$$

$$-\int p_q(u)du\ \underset{\gamma\to 0}{\Rightarrow}\ \left\{\begin{array}{l}\varepsilon\exp\{-\varepsilon u\}(u+1/\varepsilon),\quad u<c;\\ 0,\quad u\geq c.\end{array}\right.$$

Thus, as in 4f) it is received "finite memory" on an interval $(0,c)$, but contributions in NSO, amendments to Zubarev's NSO are finite and at $\langle\Gamma\rangle\to\infty$. "Finite memory" is possible and for "usual" functions of distribution by $u\geq c$ and for fractional distributions of type of Pareto distribution. So, for

4i).

$$p_q(u)du=\left\{\begin{array}{l}\varepsilon\exp\{-\varepsilon u\},\quad u<c;\\ \frac{b\exp\{-\gamma u\}}{(1+au)^m},\quad u\geq c,\end{array}\right.$$

from a normalization condition

$$b=\frac{a\exp\{-\varepsilon c\}\exp\{-\gamma a\}}{(\gamma/a)^{1-m}\Gamma(1-m,\gamma/a)},$$

$\Gamma(,)$ is incomplete gamma function,

$$\langle\Gamma\rangle=\frac{1}{\varepsilon}[1-\exp\{-\varepsilon c\}(1+\varepsilon c)]+\exp\{-\varepsilon c\}[\frac{\gamma\Gamma(2-m,\gamma/a)}{a^2\Gamma(1-m,\gamma/a)}-1].$$

The first moment $\langle\Gamma\rangle\to\infty$ at $a\to 0$. In this case $b\to\gamma$, and

$$p_q(u)du\ \underset{a\to 0}{\Rightarrow}\ \left\{\begin{array}{l}\varepsilon\exp\{-\varepsilon u\},\quad u<c;\\ \gamma\exp\{-\varepsilon c\}\exp\{-\gamma u\},\quad u\geq c.\end{array}\right.$$

$$-\int p_q(u)du=\left\{\begin{array}{l}\exp\{-\varepsilon u\},\quad u<c;\\ -\gamma\exp\{-\varepsilon c\}\int\frac{\exp\{-\gamma u\}du}{(1+au)^m},\quad u\geq c;\end{array}\right.$$

$$-\int p_q(u)du\ \underset{a\to 0}{\Rightarrow}\ \left\{\begin{array}{l}\exp\{-\varepsilon u\},\quad u<c;\\ \exp\{-\gamma u\}\exp\{-\varepsilon c\},\quad u\geq c,\end{array}\right.$$

as in a case (14), i.e. additives do not address in a zero at $u\geq c$. The value $\langle\Gamma\rangle\to\infty$ and at $\gamma\to\infty$. In this case $p_q(u)=0$ at $u>c$, memory is finite.

5 The conclusion

As it is specified in work [8], existence of time scales and a stream of the information from slow degrees of freedom to fast create irreversibility of the macroscopical description. The information continuously passes from slow to fast degrees of freedom. This stream of the information leads to irreversibility. The information thus is not lost, and passes in the form inaccessible to research on Markovian level of the description. For example, for the rarefied gas the information is transferred from one-partial observables to multipartial correlations. In work [3] values $\varepsilon = 1/\langle\Gamma\rangle$ and $p_q(u) = \varepsilon \exp\{-\varepsilon u\}$ are expressed through the operator of entropy production and, according to results [8], - through a stream of the information from relevant to irrelevant degrees of freedom. Introduction in NSO function $p_q(u)$ corresponds to specification of the description by means of the effective account of communication with irrelevant degrees of freedom. In the present work it is shown, how it is possible to spend specification the description of effects of memory within the limits of method NSO, more detailed account of influence on evolution of system of quickly varying variables through the specified and expanded kind of density of function of distribution of time the lived system of a life.

In many physical problems finiteness of lifetime can be neglected. Then $\varepsilon \sim 1/\langle\Gamma\rangle \to 0$. For example, for a case of evaporation of drops of a liquid it is possible to show [33], that non-equilibrium characteristics depend from $\exp\{y^2\}; y = \varepsilon/(2\lambda_2)^{1/2}, \lambda_2$ is the second moment of correlation function of the fluxes averaged on quasi-equilibrium distribution. Estimations show, what even at the minimum values of lifetime of drops (generally - finite size) and the maximum values size $y = \varepsilon/(2\lambda_2)^{1/2} \leq 10^{-5}$. Therefore finiteness of values $\langle\Gamma\rangle$ and ε does not influence on behaviour of system and it is possible to consider $\varepsilon = 0$. However in some situations it is necessary to consider finiteness of lifetime $\langle\Gamma\rangle$ and values $\varepsilon > 0$. For example, for nanodrops already it is necessary to consider effect of finiteness of their lifetime. For lifetime of neutrons in a nuclear reactor in work [3] the equation for $\varepsilon = 1/\langle\Gamma\rangle$ which decision leads to expression for average lifetime of neutrons which coincides with the so-called period of a reactor is received. In work [35] account of finiteness of lifetime of neutrons result to correct distribution of neutrons energy.

Use of distributions (5), (6), (9), (13) and several more obvious forms of lifetime distribution in quality $p_q(u)$ leads to a conclusion, that the deviation received by means of these distributions values $\ln \varrho(t)$ from $\ln \varrho_{zub}(t)$ is no more $\varepsilon \sim 1/\langle\Gamma\rangle$. Therefore in expression (1) additives to the result received by Zubarev, are proportional ε. This result corresponds to mathematical results of the theory asymptotical phase integration of complex systems [27] according to which distribution of lifetime looks like $p_q(u) = \exp\{-\varepsilon u\} + \lambda\varphi_1(u) + \lambda^2\varphi_2(u) + \ldots$, where the parameter of smallness λ in our case corresponds to value $\varepsilon \sim 1/\langle\Gamma\rangle$. Generally the parameter λ can be any.

For distributions of kind (11), (14), (15), having a various form for different times, additives to Zubarev's NSO are distinct from zero and for infinitely large systems with infinitely large lifetimes. For some distributions the effect of "finite memory" when only the limited interval of the past influences on

behaviour of system is observed.

Probably, similar results will appear useful, for example, in researches of small systems. All greater value is acquired by importance of description of the systems in mesoscopical scales. A number of the results following from interpretation of NSO and $p_q(u)$ as density of lifetime distribution of system [3], it is possible to receive from the stochastic theory of storage [34] and theories of queues. For example, in [34] the general result that the random variable of the period of employment (lifetime) has absolutely continuous distribution $p_q(u) = g(u, x) = xk(u - x, u), u > x > 0$ is resulted; $g(u, x) = 0$ in other cases, where $k(x, t)$ is absolutely continuous distribution for value $X(t)$ - input to system.

The form of distribution chosen by Zubarev for lifetime represents limiting distribution. The choice of lifetime distribution in NSO is connected with the account of influence of the past of system, its physical features, for the present moment, for example, with the account only age of system, as in Zubarev's NSO [1, 2, 3] at $\varepsilon > 0$, or with more detailed characteristic of the past evolution of system. The received results are essential in cases when it is impossible to neglect effects of memory when memory time there is not little. The analysis of time scales as it is noted in [8], it is necessary to spend in each problem.

Generalization of the received results on wider (generally any distributions $F(x)$) classes of distributions is received in work [12] with use of methods of the renewal theory. In [12] it is shown, that the normalizing random variable $(t - t_0)/t$ at $t \to \infty$ has limiting density $g_\alpha(x) = (sin\pi\alpha/\pi)x^{-\alpha}(1 - x)^{\alpha-1}, 0 < \alpha < 1, x \in [0, 1]$, connected with functions of distribution $F(x)$, having correctly varying tails, $1 - F(x) = x^{-\alpha}L(x), 0 < \alpha < 1$, where $L(tx)/L(t) \to 1$ at $t \to \infty$. Average value $\langle\Gamma\rangle/t = (\alpha - 1)sin\pi\alpha/sin\pi(\alpha - 1)$. As $\langle\Gamma\rangle/t = \delta$ is small size values α are close to unit and $\delta \approx \sin\pi\alpha/\pi$. At $\alpha \approx 1 - \delta, sin(1 - \delta)\pi = sin\pi\delta = \delta\pi - (\delta\pi)^3/3 + ... \approx \pi\delta$, we receive identity. At $\alpha \approx 1 - \delta$ distribution $g_\alpha(x) \approx \delta(1-x)^{-\delta}/x^{1-\delta}$ behaves in the similar image with $\varepsilon \exp\{-\varepsilon x\}$ at $\varepsilon \sim \delta$, differing at $x \to 0$. In this case universal distribution also is characterized only by one parameter α, but the limiting situation $t \to \infty$ and influence of tails of distribution, probably, not absolutely full describes past influence on the present as the near moments of time are thus more significant, with the memory which has not gone yet.

In Prigogine's work [6] of function of distribution, evolving in course of time in accordance with the laws of mechanics, through transformation a distribution function is put in accordance, the evolution of which is described by probabilistic rule. The role of such transformation in the method of NSO plays averaging on the density of distribution of time by the spent system of life.

If type of source in Liouville equation for a non-equilibrium statistical operator in the form of Zubarev [2] it is possible to confront with a linear relaxation source in Boltzmann equation, more difficult types of sources, got from other distributions for lifetime of the system, it is possible to compare to more realistic type of integral of collisions, that is explained by the openness of the system, by its co-operation with surroundings and finiteness of lifetime of the system, and also coarsening for physically infinitely small volumes.

The problems of correlation of spatial and temporal descriptions are inter-

esting. So, spatial description in works of Klimontovich [13], smoothing out on physically infinitely small volume, yields to results similar with the results of Zubarev, to the same expression for a source. Account of influence of surroundings, interaction with other systems (spatial task) also brings to the similar results. So, in works of MacLennan, short exposition of which and accordance with Zubarev's works is given in the Appendix 2 to the book of Zubarev [2], a source in Liouville equation become formed by thermodynamical variables - temperature, chemical potential and speed characterizing surroundings, not details of his microscopic state. If, as in works [4, 5] to conduct replacement of temporal argument of thermodynamical variables and to increase them on the proper "weigth functions" (which in [1] is interpreted as densities of distribution of time by the spent system of life), we will get stated in the offered work results.

Correlations between the exponential damping (3) and nonexponential functions just for small, non-Markov time scale factors, considered for a quantum case in works [36].

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